The existence of two stages in the transition to three-dimensionality of a cylinder wake

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The transition to three-dimensionality in the near wake of a circular cylinder involves two successive transitions, each of which corresponds with a discontinuity in the Strouhal–Reynolds number relationship. The first discontinuity [between Reynolds numbers (Re) of 170 to 180] is associated with the inception of vortex loops, and it is hysteretic. The second discontinuity (between Re = 230 to 260) corresponds with a change to a finer-scale streamwise vortex structure. At this discontinuity there is no hysteresis, and it is suggested that two modes of vortex shedding alternate in time.

A great deal of attention has been paid, in previous work, to the problem of laminar vortex formation in the wake of a circular cylinder. However, very few of these studies extended their investigations to include the three-dimensional transition of the near wake, which occurs at slightly higher Reynolds numbers. This is surprising, as there are a number of interesting features of this transition that have largely remained unexplored.

The first extensive wake measurements over a wide range of Reynolds numbers (Re) were undertaken by Roshko[1] in 1954, who defined first a "stable" (laminar) vortex shedding regime up to $Re \approx 150$, followed by a region of transition, preceding the "irregular" regime, which was reached for $Re > 300$. The "irregular" regime was so called because of the irregularities present in the periodic wake velocity fluctuations. Since that time, very few measurements have been made in the transition regime, although a small amount of flow visualization has been done. Hama[2] and Gerrard[3] both observed a waviness in the primary wake vortices during shedding. Gerrard described the streamwise elongation of these waves as "fingers of dye," but did not associate them with the development of streamwise vorticity. It was earlier suggested by Roshko[1,4] that "transition to turbulence" must occur in the separating shear layers before the vortices fully form and break away; otherwise it will not occur. High-frequency oscillations were later detected in the separating shear layer by Bloor.[5] These oscillations were attributed to the two-dimensional instability of the layer, before it rolls up into the primary wake vortices. Wei and Smith[6] observed that "secondary" vortices were associated with these high-frequency oscillations, and hypothesized that it is the three-dimensional stretching of these "secondary" vortices that causes streamwise vortices to appear in the wake of a cylinder. Recently, much attention has been given to the development of streamwise vortices in other flows such as in free shear layers (for example, Bernal and Roshko[7] and Corcos and Lin[8]), and also in the "unseparated" wake that forms behind a splitter plate (Meiburg and Lasheras[9]).

In the present study, three-dimensional structures in the wake were observed when $Re > 178$. The "secondary" (or Kelvin–Helmholtz) vortices in the shear layer (referred to earlier) begin to form at Reynolds number of around 1000, and therefore cannot explain the onset of vortex loops and streamwise vortices that are found here at much lower Reynolds numbers. It is actually the deformation of the primary vortices themselves, during the process of vortex shedding, that creates the three-dimensional loops and streamwise vortices. It is also found here that the transition regime is characterized by *two* discontinuous changes in the Strouhal–Reynolds number relationship, as shown in Fig. 1. The two discontinuities are associated with the inception of two different scales of three-dimensional structures in the near

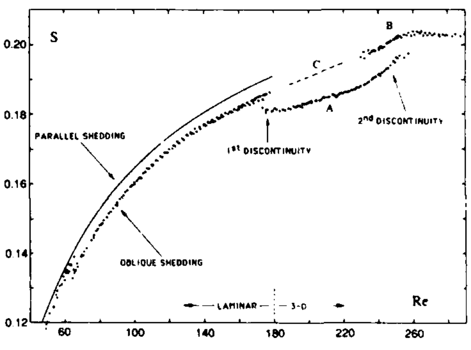

FIG. 1. Strouhal numbers (S) versus Reynolds numbers (Re) for three different cylinders. The two discontinuities are associated with the inception of different forms of three-dimensional structures. We see ○, $D = 0.104$ cm ($L/D = 140$); ●, $D = 0.061$ cm ($L/D = 200$); +, $D = 0.051$ cm ($L/D = 240$); D = cylinder diameter; L = cylinder length.

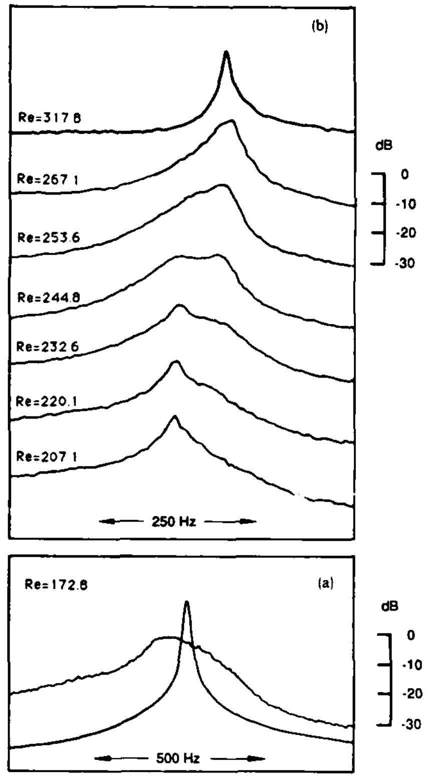

FIG. 2. Spectra of velocity fluctuations in the cylinder wake. (a) Spectra at first discontinuity. Here Re = 172.8, $S = 0.1840$, $f = 1305$ (sharp spectrum); $S = 0.1802$, $f = 1277$ (broad spectrum); $D = 0.061$ cm. (b) Spectra across second discontinuity. Each spectra is centered at different frequencies. Re = 207.1, $f = 547$; Re = 220.1, $f = 598$; Re = 232.6, $f = 641$ and 669; Re = 244.8, $f = 693$ and 719; Re = 253.6, $f = 729$ and 753, Re = 267.1, $f = 795$; Re = 317.8, $f = 952$. $D = 0.104$ cm. Here f = frequency of peaks in the spectra (measured in hertz).

wake. Interestingly, most of the scatter in Roshko's earlier measurements[1] of Strouhal number is to be found in the vicinity of either of these two discontinuities.

The present measurements have been made with three cylinders (of diameters 0.051, 0.061, and 0.104 cm) in a 6 in. circular test section of an open jet wind tunnel. The turbulence level was close to 0.1%, with flow uniformity better than 0.3%. Endplate disks of 20:1 diameter ratio were used, with cylinder length to diameter ratios up to 240. A good deal of effort was taken to isolate the cylinders from the tunnel, and to damp out any cylinder vibrations. A spectrum analyzer was used to measure frequencies from a hot wire in the wake of the cylinder.

In the present work, the laminar flow regime is found to extend from Re = 49–178, but in Fig. 1 we can see that in this range of Re there are two Strouhal curves at different levels. The lower curve with the data points corresponds with the phenomenon of oblique vortex shedding, as demonstrated by Williamson,[10] and the small discontinuity at Re = 64 is caused by a change in the mode of the oblique shedding. However, it was found that if parallel shedding was "induced" to occur by manipulating the end conditions, then a completely continuous and somewhat higher curve can be obtained, which is also shown in Fig. 1 by the solid curve. At every point on this curve the spectra are sharply peaked [as shown typically by the sharp spectrum in Fig. 2(a)].

In the transition regime, an interesting aspect of the Strouhal–Reynolds number relationship is the observation of two distinct discontinuities (as mentioned earlier), which can be seen clearly in Fig. 1. Some confidence in these results is assured by the fact that there is close agreement between experiments using three different cylinders. The first discontinuity is associated with a change from periodic, laminar vortex sheddding to shedding which involves the formation of vortex loops (as shown later in Fig. 3). The second discontinuity is related to a change in the three-dimensional structure of the wake from the vortex loops to finer-scale streamwise vortices (these can be seen in Fig. 4). The behavior of the spectra corresponding to each of these discontinuities is markedly different.

In the case of the first discontinuity, there is a small overlap of the Strouhal curves near Re $\simeq$ 180, and also a hysteresis, which is demonstrated in Fig. 2(a). The sharply peaked spectrum was obtained by *increasing* Re up to 172.8, and this spectrum reflects the presence of laminar vortex shedding; in this case we are on the (upper) laminar Strouhal curve (see Fig. 1). The broad spectrum in Fig. 2(a) was the result of *decreasing* Re down to 172.8, and it is associated with the less-ordered velocity fluctuations caused by the inception of vortex loops; we are now on the lower Strouhal curve marked A (in Fig. 1).

The second discontinuity is distinctly different from the first. It not only extends over a larger range of Re (from 230–260), but involves the existence of two peaks in the spectra simultaneously, as shown in Fig. 2(b). Consequently, the flow behavior does not exhibit a hysteresis, as found for the first discontinuity. We can see that there is a gradual shift of energy from the lower peak to the upper peak of the spectra, as Re is increased (i.e., moving upward in this figure). This corresponds, in Fig. 1, to a transfer of energy from the Strouhal frequencies associated with curve A to those frequencies associated with curve B. By the time Re = 260, it is difficult to determine a lower peak (corresponding to curve A) in the spectrum any more, and the spectra gradually become sharper as Re is increased to around 350. The presence of two peaks occurring simultaneously in the spectra might be assumed to be due to the presence, along the cylinder span, of cells of different frequency. By traversing a hot wire in the spanwise direction, it was concluded that this was not the case. Instead, it is suggested that the lower-frequency mode of shedding alternates in time with the upper-frequency mode; the flow is neither locked into one mode nor the other.

Experiments to visualize the three-dimensional structures, which are associated with the above wake measurements, were conducted in the new X-Y Towing Tank at the Graduate Aeronautical Laboratories, California Institute of

Technology. Some results of flow visualization in the tank (using dye washed off the surface of a cylinder of diameter 1.27 cm) are shown in Figs. 3 and 4. The sequence of photos in Fig. 3 shows the development of vortex loops, which were observed when the Strouhal frequency lay along curve A in Fig. 1 (i.e., when Re is roughly between 178 and 260). It was found that the characteristic wavelength between these vortex loops was around three cylinder diameters. In the photographs, the camera is fixed with respect to the vertical cylinder, whose edge is seen at the extreme right of each picture. Only a small portion of the span is visualized (about three diameters in length). The process of loop generation is self-sustaining in that there is a feedback from one loop to the next, so that a whole string of vortex loops form at the same spanwise position. This can be understood in terms of the induced velocity of vortex loops, although such details are not presented here. Referring to the vortex loop marked with a star in the photograph sequence, we can see that the vortex loops become highly stretched, so that the two sides of each loop evolve into a pair of contrarotating streamwise vortices. (Further evidence to support this description has been obtained from other techniques, and will be included in a forthcoming paper.) From the present visualization, it was found that the primary vortices roll up first and then deform, during the process of shedding, to create the vortex loops. It is thus not necessary for the shear layer instability vortices (the "secondary" vortices referred to earlier) to be present for streamwise vortices to be formed.

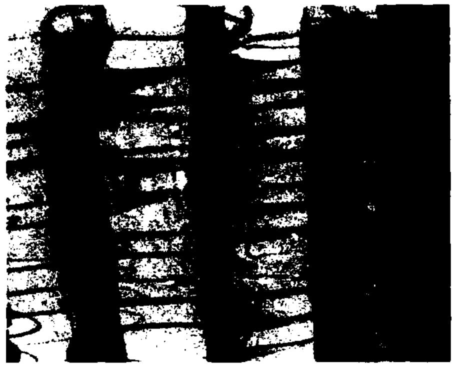

FIG. 4. Generation of fine-scale streamwise vortices after the second Strouhal discontinuity, Re = 285. (This corresponds with Strouhal numbers along curve B in Fig. 1.) Cylinder is vertical and at extreme right, and flow is from right to left. Length of span visualized is about 6.5 diam.

At higher Reynolds numbers, when the Strouhal frequencies lie on curve B in Fig. 1 (i.e., after the second discontinuity), finer-scale streamwise vortices are formed, as shown in Fig. 4. The primary vortices again deform during shedding, but with a markedly smaller spanwise wavelength of around one diameter. In the present case of a *separated* wake, the streamwise vorticity originates mainly from the deformation of the primary "core" vorticity, which is *subsequently* stretched in the "braid" regions between the primary vortices. This is rather different from the origin of streamwise vortices in an *unseparated* wake, described by Meiburg and Lasheras,[9] where the streamwise vortices originate *in the first instance* from vorticity in the "braid" region.

In conclusion, the transition regime involves *two* successive changes in the mode of vortex formation, as Re is increased. First, there is a change from laminar vortex shedding to a form of shedding which involves the generation of vortex loops. Second, there is a change in the type of three-dimensional structures in the near wake; from the vortex loops to finer-scale streamwise vortices. Corresponding with each change is a discontinuity in the Strouhal–Reynolds number relationship. The character of each discontinuity is distinctly different: the first discontinuity is hysteretic, while the second is not. Finally, one might speculate, from the form of the Strouhal curves in Fig. 1, that the Strouhal curve B is actually an extension of the laminar vortex shedding curve, as suggested by the dashed curve marked C. Corresponding with such a curve one might expect a single transition, with the formation of the fine-scale streamwise vortices (appearing at about Re ≈ 250) following directly from the laminar vortex shedding regime. It seems possible that the Strouhal curve A found in experiments is a more "stable" route in the transition to three-dimensionality than curve C, and consequently there are *two* stages in the transition process rather than simply one. The present work is part of a

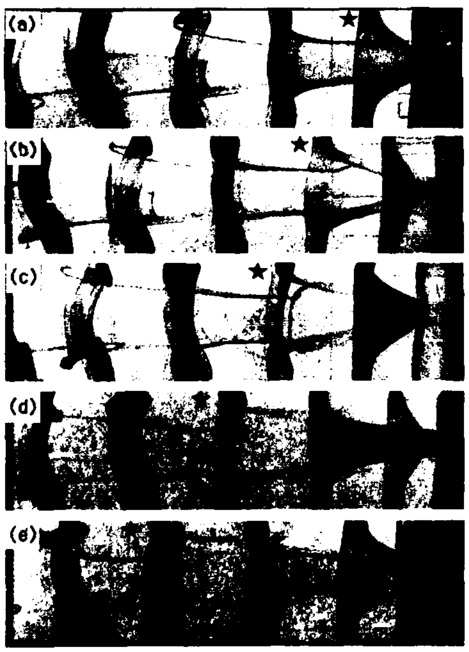

FIG. 3. Generation of vortex loops and pairs of streamwise vortices after first Strouhal discontinuity, Re ≃ 180. (This corresponds with Strouhal numbers along curve A in Fig. 1.) Cylinder is vertical and at extreme right of each photograph, and flow is from right to left. Length of span visualized is about 3 diam.

more comprehensive piece of research which will appear in a forthcoming paper.

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